JOURNAL OF AIRCRAFT
Vol. 48, No. 2, March–April 2011

Uncertainty Importance Measure by Fast Fourier Transform for Wing Transonic Flutter

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DOI: 10.2514/1.C031038

Considering the natural frequency, parameters of gravity center, and mass ratio of aircraft wing as random parameters, the global sensitivity, also named as uncertainty importance measure, is analyzed on the fast Fourier transform. The most crucial and difficult problem of importance measure is how to obtain the unconditional and conditional probability density function or failure probability of the model rapidly and properly. The fast Fourier transform technique can estimate the probability density function and cumulative distribution function of the structural response efficiently and robustly, so two moment-independent importance measure indexes (including the importance measure of basic random variable on the probability distribution of response and the importance measure of basic random variable on the failure probability) are solved by use of the fast Fourier transform technique. For the two-dimensional aircraft wing transonic flutter problem, reduced order modeling method on computational fluid dynamics is used to construct the aerodynamic state equations. Coupling structural state equations with aerodynamic state equations, the state equations of aeroelasticity system can be obtained, on which the limit state function of flutter is founded by considering the critical velocity, which is solved by the eigenvalue of the state matrix, satisfying the requirement. For the aeroelastic flutter response models of a two-dimensional wing without flap and with a flap, two importance measure indexes can quantificationally reflect the influence of the random variables on the structural response. Comparing with the importance measure results of Monte–Carlo simulation, those of fast Fourier transform are higher in efficiency with acceptable precision.

I. Introduction

AEROELASTIC analysis is an interdisciplinary subject of aerodynamic load, elastic force, and inertia force, and it is very important in the aircraft design [1,2]. Flutter is a typical aeroelastic problem in the aircraft design, and it can lead to aviation tragedy due to structural vibration divergence in a short time. General studies about the flutter issue are based on the deterministic model, which assumes all the structural parameters as complete determinacy. However, there are uncertainties of manufacture, machining, meterage, environment, and working load, etc., in the engineering practice, so the deterministic analysis cannot provide enough information about the flutter responses. To study the issue of wing flutter more rationally under the random uncertainties, the uncertainty analysis must be introduced [3,4]. Considering the uncertainty parameters as basic random variables, the sensitivity analysis can quantificationally reflect the influence of the random variables on the flutter response, which can help to identify the important parameters and guide the optimization design.

For the reliability problem with basic random variables, the statistic characteristics (i.e., the failure probability, the statistical moment, the probability density function [PDF], and the cumulative distribution function [CDF], etc.) and reliability sensitivity of structural response are studied in the scores of year. The reliability sensitivity is defined as the partial derivative of failure probability or reliability with respect to the basic random variable at the nominal value (usually at the mean value). This reliability sensitivity cannot reflect the influence of the variability of other random variables on the reliability sensitivity of some one, so it is a local sensitivity substantively [5–8]. Comparing with the local sensitivity, the contribution of the randomness of input variables to that of output response, named as uncertainty importance measure (UIM), is the global sensitivity. Saltelli [9] and Borgonovo [10,11] pointed out that the importance measure should be *global*, *quantitative*, *model free*, and *moment-independence*. For the four requirements of the importance measure, many scholars presented their measure indexes [11–15].

In the process of obtaining the moment-independent UIM of basic random variable on the probability distribution of response δ [11] and the UIM of basic random variable on the failure probability η [15], the most crucial and difficult problem is how to obtain the unconditional and conditional PDF or failure probability of the model rapidly and properly. Currently, the simple method of solving the PDF or failure probability of structural response is the direct Monte–Carlo simulation (MCS). By random drawing a large number of structural response samples, the histogram analysis is employed to identify the characteristic parameters and fit the PDF of response. The major disadvantage of MCS is that a huge amount of calculation is needed for the high-precision approximation at the tail distribution of response, which is important for the failure probability. In the reliability design we would pay more attention to the input, which is significantly important on the failure probability. Hence, the importance measure η, which describes the effect of the basic variable on the failure probability, is a crucial factor in the reliability analysis and design, and the importance measure of the basic variable on the distribution of model response cannot substitute the importance measure of the basic variable on the failure probability. Additionally, to a complex performance function, it is not easy to obtain or approximate the whole probability distribution, but there are many available methods to solve its failure probability.

In virtue of probability conservation law, the theoretical foundation of the fast Fourier transform (FFT) method involves that 1) the PDF and its characteristic function are a pair of Fourier transforms, and 2) the characteristic function of the sum of statistically independent random variables is given by the product of the characteristic functions of each random variable. Then the FFT method can estimate of the PDF and CDF of the random variable defined as the linear function of the independent random variables and can be applied when either the PDF or the characteristic function of each basic random variable is known [16–18]. It should be noted that the method is very efficient with the use of FFT and also has robustness.

Received 25 March 2010; revision received 18 October 2010; accepted for publication 16 November 2010.

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When the limit state function is nonlinear or implicit function of basic random variables, the second-order approximation at the most probable point (MPP) [19], quadratic polynomial response surface without the cross terms [20,21] and two-point adaptive nonlinear approximation (TANA2) [22,23] can be used to give a closed-form equation in a separable form of random variables, which is required for the FFT method. In this paper, FFT technique is used to calculate the UIM δ and η rapidly and robustly, named as FFT-based importance measure.

For flutter analysis, coupling structural equations with Euler/NS-based unsteady computational fluid dynamics (CFD) algorithm, the structural response can be predicted in time domain with the fewest assumptions about the characteristics of the flowfield. However, the challenges offered by this kind of aeroelastic analysis are those of computational time and their low effectiveness in the flutter elimination and parametrical design environments. To solve the contradiction between computational efficiency and computational quality, many researchers turned to use CFD-based unsteady aerodynamic reduced order modeling (ROM) method to improve the aeroelastic computational efficiency in the last decade [24–26]. Reference [26] compared the efficiency of ROM-based method and CFD direct simulation method, the efficiency of ROM-based method can be improved by 1–2 orders of magnitude with accuracy still be retained. References [27,28] used CFD-based ROM to perform aeroservoelastic analysis, active flutter suppression and flutter analysis at high angle of attack. Based on the available methods of the flutter analysis, the safety margin of flutter reliability analysis is founded by use of the critical velocity of flutter.

This paper devotes to analyze the reliability and global sensitivity for the transonic flutter by use of the FFT technique and uses the results of the reliability and global sensitivity analysis to investigate the effects of various parameters on the flutter responses. And the failure probability and UIM indexes is analyzed with the randomness of frequency. In this work, the UIM of basic random variable on the distribution of response δ and the UIM of random variable on the failure probability η are introduced in Sec. II. Section III gives the concept and implementation of FFT-based importance measure. Several numerical examples are used to demonstrate the advantages of FFT-based importance measure, and then the transonic flutter of two-dimensional aircraft wing models are used to analyze the UIM of basic random variables in Sec. IV. Finally, Sec. V concludes with a summary of the presented FFT-based importance measure.

II. UIM of the Basic Random Variable

A. UIM of Basic Random Variable on the Distribution of Response δ [11]

Given a performance response function $Z = g(X_1, X_2, \cdots, X_n)$, where Z is the model output, and $\boldsymbol{X} = \{X_1, X_2, \cdots, X_n\}$ are input basic random variables. Denote $f_Z(z)$ and $f_{Z|X_i}(z)$ as the unconditional and conditional PDF, respectively, $f_{Z|X_i}(z)$ can be obtained by assuming the input variable X_i at a fixed realization value x_i^*. The absolute value of the difference between $f_Z(z)$ and $f_{Z|X_i}(z)$ can represent the effect of X_i on the distribution of Z when X_i takes x_i^*. When z takes value from $-\infty$ to $+\infty$, the cumulative effect of X_i on the distribution of Z can be measured by the $s(X_i)$, which is given by the following integral:

$$s(X_i) = \int_{-\infty}^{+\infty} |f_Z(z) - f_{Z|X_i}(z)| \,\mathrm{d}z \tag{1}$$

Because X_i is a random variable, with PDF denoted by $f_{X_i}(x_i)$, the average effect of X_i on the distribution of response Z could be described by the expected value of $s(X_i)$ with PDF $f_{X_i}(x_i)$, i.e., $E_{X_i}[s(X_i)] = \int_{-\infty}^{+\infty} f_{X_i}(x_i) s(X_i)\,\mathrm{d}x_i$. Scaling the importance measure of the input basic variable on the distribution of the model output response Z within the interval [0,1], the following quantity δ_i as a moment-independent measure of the input basic variable X_i on the distribution of the output Z can be constructed

$$\delta_i = \frac{1}{2} E_{X_i}[s(X_i)] = \frac{1}{2}\int_{-\infty}^{+\infty} f_{X_i}(x_i) s(X_i)\,\mathrm{d}x_i \tag{2}$$

B. UIM of Basic Random Variable on the Failure Probability η [15]

In reliability model $Z = g(\boldsymbol{X})$, we denote P_{f_Z} as the unconditional failure probability, i.e., $P_{f_Z} = P\{g(\boldsymbol{X}) \le 0\}$. When the input basic variable X_i is fixed at one given value x_i^*, the conditional failure probability $P_{f_{Z|X_i}}$ can be obtained. Eliminating the uncertainty of X_i has effects on the failure probability, and this effect can be measured by the difference between P_{f_Z} and $P_{f_{Z|X_i}}$. Similar to the importance measure δ, the moment-independent importance measure η is proposed to reflect effect of the input basic variable X_i on the failure probability when X_i takes its values according to its PDF $f_{X_i}(x_i)$, it is defined as

$$\eta_i = \frac{1}{2} E_{X_i}[|P_{f_Z} - P_{f_{Z|X_i}}|] = \frac{1}{2}\int_{-\infty}^{+\infty} |P_{f_Z} - P_{f_{Z|X_i}}| \cdot f_{X_i}(x_i)\,\mathrm{d}x_i \tag{3}$$

From the definition of two random uncertainty importance measures, it can be found that they both represent the effect of the basic random variable on the distribution of the performance response function, the difference between them is that importance measure δ represents the effect of the basic variable on the whole distribution of the model output, while the importance measure η represents the effect of the basic variable on the failure domain of the model output. In other words, the latter pays more attention to the tail distribution of the model output. So the regions they discuss and the scenarios they are applied for are different.

III. FFT-Based Solutions for Two Importance Measures

Taking into account that 1) the PDF and its characteristic function are a pair of Fourier transforms, and 2) the characteristic function of the sum of statistically independent random variables is given by the product of the characteristic functions of each random variable, FFT probability analysis method can estimate of the CDF of the random variable defined as the linear function of the independent random variables and can be applied when either the PDF or the characteristic function of each basic random variable is known. When the limit state function is nonlinear or implicit function of basic random variables, we should give a closed-form equation in a separable form of random variables firstly [19–23], and then the transformation technique is employed to obtain the PDFs of intervening variables. Finally, the FFT method can be used to analyze the PDF and CDF robustly.

A. Probability Analysis Method Using FFT

We know that the PDF $f_X(x)$ of random variable X and its characteristic function $\varphi_X(\theta)$ are a pair of Fourier transforms. For the linear function, the characteristic function of function response can be obtained by using the following properties:

1) The characteristic function of variable $Z = a \cdot X + b$ is expressed as $\varphi_Z(\theta) = e^{ib2\pi\theta} \cdot \varphi_X(a\theta)$.

2) The characteristic function of variable Z, which is the sum of statistically independent random variables $X_1, X_2, \cdots, X_n$, is given by the product of the characteristic function of each random variable $\varphi_{X_1}(\theta), \varphi_{X_2}(\theta), \cdots, \varphi_{X_n}(\theta)$ as $\varphi_Z(\theta) = \varphi_{X_1}(\theta)\varphi_{X_2}(\theta)\cdots\varphi_{X_n}(\theta)$.

The closed form of the characteristic function of a random variable is available for limited cases such as a random variable described by a normal distribution function and a gamma distribution function. In structural reliability analysis, however, random variables are often described by some else distribution functions. Although the characteristic functions of these random variables cannot be expressed in closed forms, they can be evaluated efficiently using FFT with discretization of the PDF [16–18]. Once the PDF of the limit-state is obtained in the form of discrete values, numerical integration can be used to determine the CDF plot and the failure probability. This plot enables the estimation of failure probability at different limiting values of the limit-state without performing another analysis.

B. FFT-Based Solutions for Two Moment-Independent Importance Measures

By use of FFT methods, the numerical solution of the unconditional PDF $f_Z(z)$ of the performance response function $Z = g(X_1, X_2, \cdots, X_n)$ can be obtained, and the numerical solution of the conditional PDF $f_{Z|X_i}(z)$ with X_i fixed at the given realization. Substituting them into Eq. (2), one can obtain the importance measure of input basic random variables on distribution of Z. Similarly, once the numerical solution of $f_Z(z)$ and $f_{Z|X_i}(z)$ are obtained, the unconditional failure probability $P_{f_Z} = \int_{-\infty}^{0} f_Z(z)\,dz$ and the conditional failure probability $P_{f_{Z|X_i}} = \int_{-\infty}^{0} f_{Z|X_i}(z)\,dz$ can be solved conveniently. Substituting P_{f_Z} and $P_{f_{Y|X_i}}$ into Eq. (3), the importance measure η_i of the input basic variable X_i on the failure probability can be obtained.

Because the unconditional PDF and the conditional PDF of the performance response function can be solved by using of FFT method effectively, it becomes convenient to solve two moment-independent importance measures. Compared with the histogram or the density distribution function fitting methods which were used to solve the importance measure δ in the existed literature [8], FFT-based solution has advantages both in precision and efficiency. So we say it has extensive applicable field.

IV. Examples

A. Numerical Examples

The numerical examples in this subsection are used to testify the accuracy of the FFT-based method. Because the exact solutions of most conditional and unconditional PDFs are difficult to get analytically, we give several simple examples so that we can compare the results of the FFT-based solution with their exact results. In computing the importance measure δ, we take analytical results of example 1 in [9]. In computing the importance measure η, the results of MCS are taken as the references.

Example 1: The performance response function is $z = g(\boldsymbol{x}) = x_1 + x_2$, and we discuss the importance measures δ_i and η_i $(i = 1, 2)$ of two input basic variables x_1 and x_2 in two cases. Case I: $x_i \sim U(-\pi, \pi)$ $(i = 1, 2)$. Case II: $x_i \sim N(0, \pi^2/9)$ $(i = 1, 2)$. The result comparisons for two cases are listed in Table 1.

From the result of example 1 listed in Table 1, we know that the MCS and the presented method can give good results UIM δ compared with the exact results, and the presented method can give good results UIM η compared with MCS. In the results table, Monte–Carlo simulation is denoted as MCS, and the presented method is denoted as FFT. The number of transferring the performance function of MCS is $2 \times 40 \times 10^6$ (2 is the number of basic random variables, 40 is the number of fixed realization value of each variable, 10^6 is the sampling size for obtain the PDF of response), but that of the presented method is not needed for the closed-form performance function. The corresponding computation time is 2.8 days and 1.7 hours, respectively.

Example 2: The performance response function is $z = g(\boldsymbol{x}) = x_1^2 + 5x_1 + 2x_2^2 + 7x_2 + x_3^2 - 8x_3 + x_4^2 - 10x_2 - 200$, where $x_i \sim N(10, 2^2)$ $(i = 1, 2, 3, 4)$. Two importance measures δ_i and η_i of the input basic variable are listed in Table 2.

From the result of example 2 listed in Table 2, we know that the presented method can give good results compared with MCS. The number of transferring the performance function of MCS is $4 \times 40 \times 10^6$, but that of the presented method is not needed for the closed-form performance function. The corresponding computation time is 5.5 days and 2.4 hours, respectively.

Example 3: The performance response function is $z = g(\boldsymbol{x}) = 7 + x_1 + x_2 + x_3 - x_2^2 + x_3^2 - x_1x_2 + x_1x_3$ with the cross terms, where all the random variables are independent standard normal variables, i.e., $x_i \sim N(0, 1)$ $(i = 1, 2, 3)$. Two importance measures δ_i and η_i of the input basic variable are listed in Table 2.

From the results of example 3 listed in Table 2, we know that the results of the presented method are well agreed with that of MCS. The number of transferring the performance function of MCS is $3 \times 40 \times 10^6$, but that of the presented method is not needed except the limited number of transferring the performance function used for constructing the closed-form equation (about 20 sampling size). The corresponding computation time is 4 days and 4.1 hours, respectively.

Tables 1 and 2 show that results of FFT-based solution are close to the results of [9] or MCS, which testify the method presented in this contribution is feasible and accurate. Additionally, the results of example 3 in Table 2 show that the ranking of the input basic variables of two importance measures is inconsistent, as the ranking of the importance on the failure probability is x_2, x_1, and x_3, but the most unimportant variable on the whole distribution of the performance function is x_1, while variables x_2 and x_3 have the identical importance on the whole distribution of the response. Therefore, we need have a special analysis for the special problem, and the two importance measures cannot be substituted by each other.

Table 1 Computational results of two importance measures of example 1

	Case I			Case II		
	Reference	MCS	FFT	Reference	MCS	FFT
δ_1	0.33*	0.329184	0.329183	0.30*	0.303926	0.303325
δ_2	0.33*	0.329184	0.329183	0.30*	0.303926	0.303325
η_1	—	0.125625	0.124906	—	0.124341	0.124366
η_2	—	0.125625	0.124906	—	0.124341	0.124366

[a]FFT is the FFT-based solution, MCS is the MCS-based solution, * are analytical results.

B. Two-Dimensional Aircraft Wing Flutter Reliability Analysis

The important measures of the aviation engineering example presented in this paper relate to a much more complicated transonic flutter analysis of two-dimensional aircraft wing.

1. Description of Model

For the two-dimensional aircraft wing without damping, the structural motion equation can be written as

$$\boldsymbol{M}\ddot{\xi} + \boldsymbol{K}\xi = \boldsymbol{F}$$

For the 2-degree-of-freedom (DOF) problem, we have the mass matrix

$$\boldsymbol{M} = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}$$

the stiffness matrix

Table 2 Computational results of two importance measures of examples 2 and 3

		δ_1	δ_2	δ_3	δ_4	η_1	η_2	η_3	η_4
Example 2	FFT	0.15655	0.42809	0.070653	0.059193	0.0021106	0.0028067	0.0010063	0.00089206
	MCS	0.16241	0.43364	0.071580	0.059450	0.0020650	0.0027498	0.0010529	0.00088691
		δ_1	δ_2	δ_3		η_1	η_2	η_3	
Example 3	FFT	0.12194	0.27917	0.27806		0.0080864	0.016734	0.0025059	
	MCS	0.12262	0.27874	0.27887		0.0081249	0.016830	0.0024852	

Table 3 Structural parameters of model with control surface

Half chord of airfoil	$b = 0.127$ m	Dimensionless distance between the center of chord and mass center	$a_f = 0.5$
Dimensionless distance between the center of chord and rigidity center	$a = -0.5$	Mass ratio	$\gamma = 25.24$
Dimensionless distance between the mass center and rigidity center	$x_\alpha = 0.4316$	Natural frequency of bending mode (plunge frequency)	$\omega_h = 4.45$ rad/s
Dimensionless gyration radius of the airfoil gravity center with respect to the center of rigidity	$r_\alpha = 0.73014$	Natural frequency of torsion mode (pitch frequency)	$\omega_\alpha = 9.21$ rad/s
Dimensionless distance between the mass center of control surface and revolution axis	$x_\beta = 0.01985$	Natural frequency of flap mode	$\omega_\beta = 19.44$ rad/s
Dimensionless gyration radius of control surface with respect to revolution axis	$r_\beta = 0.11367$		

$$\boldsymbol{K} = \begin{bmatrix} (\omega_h/\omega_\alpha)^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix}$$

the structural generalized displacement $\boldsymbol{\xi} = [h/b, \alpha]^T$, and the generalized aerodynamic force $\boldsymbol{F} = \frac{V_f^{*2}}{\pi}[-C_l, 2C_m]^T$. For the wing with the 3-DOF problem, we have the mass matrix

$$\boldsymbol{M} = \begin{bmatrix} 1 & x_\alpha & x_\beta \\ x_\alpha & r_\alpha^2 & (a_f - a)x_\beta + r_\beta^2 \\ x_\beta & (a_f - a)x_\beta + r_\beta^2 & r_\beta^2 \end{bmatrix}$$

the stiffness matrix

$$\boldsymbol{K} = \begin{bmatrix} (\omega_h/\omega_\alpha)^2 & 0 & 0 \\ 0 & r_\alpha^2 & 0 \\ 0 & 0 & r_\beta^2(\omega_\beta/\omega_\alpha)^2 \end{bmatrix}$$

the structural generalized displacement $\boldsymbol{\xi} = [h/b, \alpha, \beta]^T$, and the generalized aerodynamic force $\boldsymbol{F} = \frac{V_f^{*2}}{\pi}[-C_l, 2C_m, 2C_n]^T$. We also have the dimensionless critical velocity of flutter $V_f^* = V_\infty/(\omega_\alpha \cdot b \cdot \gamma^{1/2})$ and the mass ratio $\gamma = m/(\pi\rho b^2)$, in which h, α and β are the denotation of plunge displacement, pitch angle, and flap deflection, respectively.

The aeroelastic response of a two-dimensional wing without flap and with a flap (denoted by model 1 and model 2, respectively) are defined as the models for the flutter analysis. Table 3 shows the structural parameters of the model as defined in Fig. 1. The type of airfoil is NACA0012.

2. *Deterministic Flutter Analysis*

A high-quality unsteady aerodynamic model is the basis for aeroelastic studies. Euler equation-based unsteady flow solver has the ability to simulate the flowfield with strong shock wave and shock wave moving. It is more efficient and maturer than the NS equation-based flowfield solver. It is used to compute unsteady aerodynamic loads based on unstructured mesh because it is applicable to the complex configuration, such as the wing-body configuration. Spatial discretization is accomplished by cell-centered finite volume formulation using center scheme. A second-order implicit scheme is used to integrate the equation in real time and the fourth Runge–Kutta time marching method is used in the pseudotime.

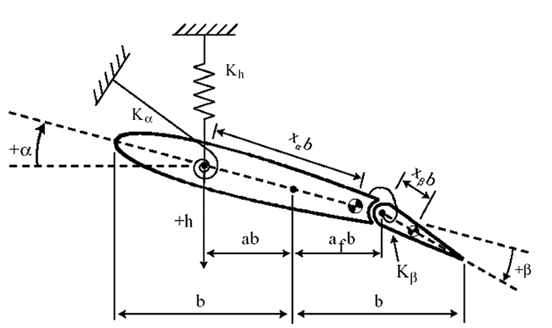


Fig. 1 Configuration of airfoil with control surface [1,30].

Giving the mode displacement (input), the generalized aerodynamic loads (output) can be computed by the mentioned flowfield solver. The state space model is

$$\begin{cases} \dot{\boldsymbol{x}}_a(t) = \boldsymbol{A}_a\boldsymbol{x}_a(t) + \boldsymbol{B}_a\boldsymbol{\xi}(t) \\ \boldsymbol{f}_a(t) = \boldsymbol{C}_a\boldsymbol{x}_a(t) + \boldsymbol{D}_a\boldsymbol{\xi}(t) \end{cases}$$

where $\boldsymbol{x}_a$ is the state vector, defined by generalized structural displacement vector $\boldsymbol{\xi}$, velocity vector $\dot{\boldsymbol{\xi}}$, and generalized aerodynamic coefficient vector $\boldsymbol{f}_a$. The detailed process can be found in [29].

By defining the structural state vector $\boldsymbol{x}_s = [\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}]^T$, the structural equations in state space is

$$\begin{cases} \dot{\boldsymbol{x}}_s(t) = \boldsymbol{A}_s \cdot \boldsymbol{x}_s(t) + q \cdot \boldsymbol{B}_s \cdot \boldsymbol{f}_a(t) \\ \boldsymbol{\xi}(t) = \boldsymbol{C}_s \cdot \boldsymbol{x}_s(t) + q \cdot \boldsymbol{D}_s \cdot \boldsymbol{f}_a(t) \end{cases}$$

where

$$\boldsymbol{A}_s = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{G} \end{bmatrix}$$

$$\boldsymbol{B}_s = \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{M}^{-1} \end{bmatrix}$$

$\boldsymbol{C}_s = [\boldsymbol{I} \quad \boldsymbol{0}]$, $\boldsymbol{D}_s = [\boldsymbol{0}]$. Defining $\boldsymbol{x} = [\boldsymbol{x}_s^T, \boldsymbol{x}_a^T]^T$ and coupling the structural state equations with aerodynamic state equations, we get the state equations of the aeroelastic system as

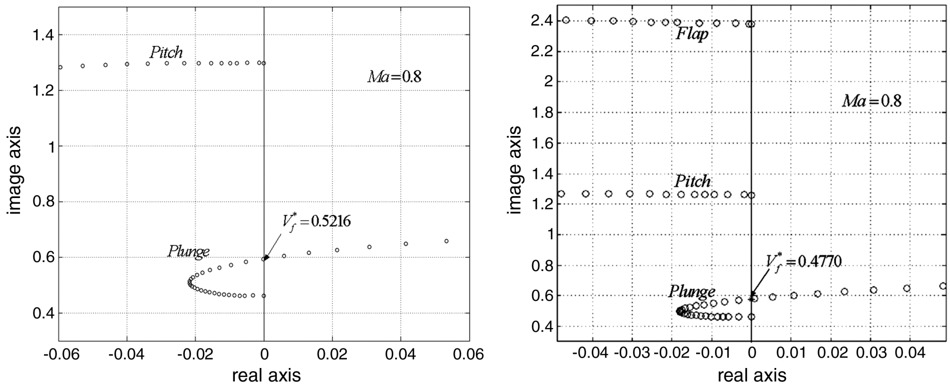
$$\dot{\boldsymbol{x}} = \boldsymbol{A} \cdot \boldsymbol{x} = \begin{bmatrix} \boldsymbol{A}_s + q \cdot \boldsymbol{B}_s\boldsymbol{D}_a\boldsymbol{C}_s & q \cdot \boldsymbol{B}_s\boldsymbol{C}_a \\ \boldsymbol{B}_a\boldsymbol{C}_s & \boldsymbol{A}_a \end{bmatrix} \cdot \boldsymbol{x}$$

By computing the eigenvalue root loci of the matrix $\boldsymbol{A}$ under different dynamic pressures, the flutter characteristics of the wing can be analyzed. According to characteristic analysis, the dimensionless critical velocity of flutter V_f^* is calculated when the Mach number of airflow speed is specified as $Ma = 0.8$. The root loci of the aeroelastic system are shown in Fig. 2 for models 1 and 2, respectively. From Fig. 2, we know that the modals including the pitch and plunge modals for model 1, considered as 2 DOF, and for model 2, the modals including the pitch, plunge, and flap modals, considered as 3 DOF.

3. *Uncertainties of Example Model*

According to the critical velocity of flutter V_f^* determined by the root locus analysis, the limit state function of wing flutter is constructed as

$$g = V_f^* - V_f$$



a) Model 1 **b) Model 2[1]**

Fig. 2 The root loci of the aeroelastic system [30].

where V_f is the dimensionless flight testing velocity and the empirical value of V_f is $V_f = V_f^*(\boldsymbol{\mu})/1.15$, and $V_f^*(\boldsymbol{\mu})$ is the critical velocity of flutter while all the random variables take their mean values.

Considering that the natural frequency, parameters of gravity center, and mass ratio are random parameters, the distribution properties of random parameters are listed in Table 3. The results of uncertainty importance measure calculated by MCS and FFT method are listed in Table 4. The results of the presented method are consistent with that of MCS. For the model 1, the number of calling the performance function of MCS is $5 \times 40 \times 10^6$, but that of the presented method is not needed except the limited number of transferring the performance function used for constructing the closed-form equation (about 35 sampling size). The corresponding computation time is 7 days and 6.8 hours, respectively. For model 2, the sampling number of MCS is $8 \times 40 \times 10^6$. About 50 sampling size are used to constructing the closed-form equation, and then FFT method need not to call the flutter analysis process. The corresponding computation time is about 10 days and 11 hours, respectively.

For the flutter model 1, taking the natural frequency $(\omega_h, \omega_\alpha)$, parameters of gravity center (x_α, r_α), and mass ratio γ as independent normal random parameters, and for the flutter model 2, taking the natural frequency $(\omega_h, \omega_\alpha, \omega_\beta)$, parameters of gravity center $(x_\alpha, r_\alpha, x_\beta, r_\beta)$ and mass ratio γ as independent normal random parameters, the uncertainty importance measures are analyzed by using the MCS and FFT techniques, respectively. When the variation coefficients of natural frequency, parameters of gravity center, and mass ratio are equal to 0.03,0.03, and 0.05, the results of UIM indexes are listed in Table 4.

The results listed in Table 4 show that the introducing of random parameters of the flap increases the UIM index η, because the randomness of the flap parameters make the failure probability increased. When the variation coefficients of natural frequency, parameters of gravity center and mass ratio are equal to 0.03,0.03, and 0.05, respectively, the essentiality order of UIM index η of random variables on the failure probability is γ,r_α, x_α, ω_α, ω_h, and that of UIM index δ of random variables on the probability density of flutter response is r_α, γ, x_α, ω_h, ω_α for the flutter model 1. For flutter model 2, the essentiality order of UIM index η is γ, r_α, x_α, ω_h, ω_β, r_β, ω_α, x_β, and that of UIM index δ is r_α, γ, x_α, ω_h, ω_β, ω_α, r_β, and x_β. Though the ranking of the input basic variables of two importance measure indexes δ and η are inconsistent, the random parameters of the flap $(x_\beta, r_\beta, \omega_\beta)$ and the frequencies $(\omega_h, \omega_\alpha)$ have less effect on the flutter reliability than the random variable x_α and r_α have at the above variation coefficient's assumption, and these conclusions are consistent with the engineering experience for the bend-torsion coupling wing [1].

When the variation coefficients of parameters of gravity center and mass ratio are equal to 0.03 and 0.05, respectively, and the coefficient of variation of natural frequencies $(\omega_h, \omega_\alpha, \omega_\beta)$ changes from 0.01 to 0.1, the failure probability and UIM indexes of the flutter reliability models are analyzed; the varying curves are shown in Fig. 3. When the variation coefficient of natural frequency $\mathrm{cov}(\omega)$ increases, the reliability of the wing structure increases, and the failure probability decreases gradually, so the important measure η of the random variables on the failure probability is gradually increasing. Furthermore, the important measure δ of natural frequencies increase and δ of parameters of gravity center and mass ratio decrease.

V. Conclusions

In engineering, research on the importance measure of the basic uncertain random variable is developed and advanced. The two moment-independent importance measures can better reflect the effect of the basic random variable. However, the exact conditional

Table 4 Results of UIM indexes ($Ma = 0.8$)

		MCS	FFT		MCS	FFT
Model 1	δ_{x_α}	0.084775	0.086914	η_{x_α} $(\times 10^{-6})$	2.4042	2.5244
	δ_{r_α}	0.32254	0.3306	η_{r_α} $(\times 10^{-6})$	2.9824	3.0566
	δ_{γ}	0.17816	0.17371	η_{γ} $(\times 10^{-6})$	3.5247	3.4635
	δ_{ω_h}	0.064625	0.063285	η_{ω_h} $(\times 10^{-6})$	1.8996	1.9708
	δ_{ω_α}	0.063215	0.06511	η_{ω_α} $(\times 10^{-6})$	1.9324	1.9049
Model 2	δ_{x_α}	0.10353	0.10146	η_{x_α} $(\times 10^{-6})$	9.3097	9.1235
	δ_{r_α}	0.25589	0.24817	η_{r_α} $(\times 10^{-5})$	1.2534	1.1695
	δ_{γ}	0.21354	0.22155	η_{γ} $(\times 10^{-5})$	1.2918	1.2302
	δ_{ω_h}	0.092655	0.094535	η_{ω_h} $(\times 10^{-6})$	8.5787	8.3861
	δ_{ω_α}	0.035441	0.03491	η_{ω_α} $(\times 10^{-6})$	3.8396	3.7820
	δ_{x_β}	0.014805	0.014739	η_{x_β} $(\times 10^{-6})$	1.7196	1.7781
	δ_{r_β}	0.035021	0.036019	η_{r_β} $(\times 10^{-6})$	4.0463	4.1616
	δ_{ω_β}	0.044649	0.045881	η_{ω_β} $(\times 10^{-6})$	5.1271	5.0529

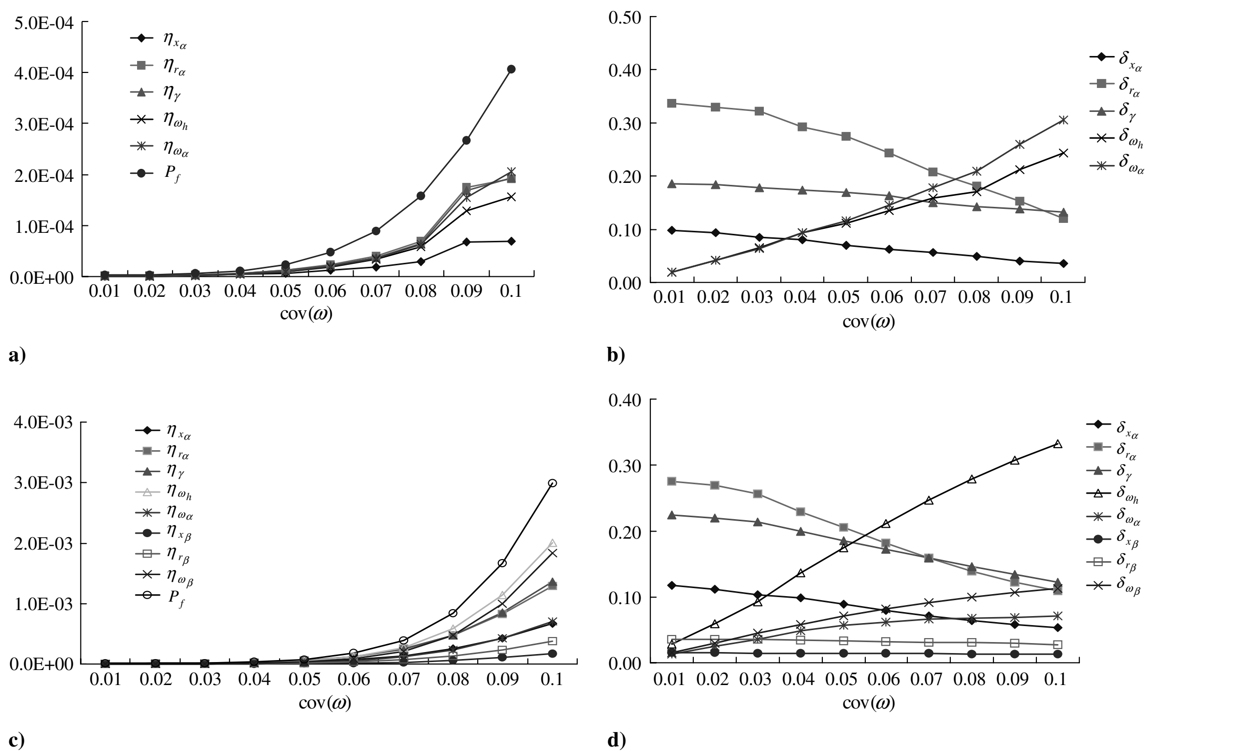


Fig. 3 Graphs of a) the varying curves of P_f and η vs cov(ω) of model 1, b) the varying curves of δ vs cov(ω) of model 1, c) the varying curves of P_f and η vs cov(ω) of model 2, and d) the varying curves of δ vs cov(ω) of model 2.

and unconditional PDF of the complex model is difficult to obtain. FFT technique can estimate the PDF and CDF of the structural response efficiently and robustly, and the FFT-based solution is proposed to solve the importance measures. The importance measure of the basic variable can give its importance ranking in the reliability model directly. The results of the numerical examples testify that the FFT-based solution not only provides an accurate enough result, but also improves the computational efficiency. The important measures analysis of two-dimensional aircraft wing flutter reliability models are carried out, and the conclusions of the rankings of the input basic variables of two importance measure indexes and the varying curves of failure probability and important measures are consistent with the engineering experience.

Acknowledgments

Supported by the National Natural Science Foundation of China (NSFC 10802063 and 50875213), the Ph.D. Program Foundation of Northwestern Polytechnical University (CX200801), and the Foundation of Fundamental Research for Northwestern Polytechnical University (NPU-FFR-JC20100201).

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